Intersecting 5-brane solution of N=1, D=10 Dual Supergravity with α' corrections

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Abstract

A vacuum solution of anomaly-free N=1, D=10 Dual Supergravity is constructed. This vacuum corresponds to the presence of two 5-branes, intersecting along M_4, and possesses N=1, D=4 supersymmetry.

1 Introduction

Various p-brane solutions in diverse supergravities were constructed in ([1] -[7]), where p-brane plays a role of a singular source of the supergravity fields. It was shown, that there are 2 types of p-brane solutions of the theory with the action:

$$I = \frac{1}{2\,k^2}\int d^D\,x\,\sqrt{-g}\,\Big(R + \frac{1}{2}\,(\nabla\varphi)^2 + \frac{e^{-\gamma\,\varphi}}{2(d+1)!}\,F^2_{(d+1)}\Big), \tag{1}$$

where $F_{(d+1)}$ - (d+1)-form, $F_{(d+1)} = dA_{(d)}$, φ - dilaton. (d-1)-brane solution with *electric* charge $Q_E \sim \int\limits_{S^{(D-d-1)}} e^{-\gamma\,\varphi\,\star}F$ is characterized by non-zero components of $A_{(d)}$ on the worldvolume of the brane, while in (D-d-3)-brane solution with *magnetic* charge $Q_M \sim \int\limits_{S^{(d+1)}} F$ only transverse components of F are "alive". $S^{(N)}$ - N-sphere at infinity, $\star$ means Hodge dual in D dimensions.

The interest in p-brane solutions of supergravities is motivated by the fact, that in the presence of the brane-source $\frac{1}{2}$ of initial supersymmetry is broken. For the review on p-branes see ([8], [9], [10]).

Using intersecting branes as a singular source of supergravity fields gives a possibility for further breaking of supersymmetry. In [11] intersecting D-branes were discussed and it was pointed out, that in some cases they preserve $\frac{1}{4}$ of initial supersymmetry.

In recent works ([12] - [17]) a rich variety of intersecting p-brane solutions in arbitrary dimensions were obtained and their supersymmetry properties analyzed, putting the main attention to solutions of D=11 Supergravity ([12], [13]) and Type II supergravities ([15], [16]).

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We want to emphasize, that so far in the literature only p-brane solutions of the supergravities, containing up to 2^d order-derivative terms in Lagrangian, were constructed.

We are interested in the solutions of anomaly-free (taking into account up to 4^{th} order-derivative terms) N=1, D=10 Dual Supergravity (**DS**), which can be viewed as field-theory limit of a Fivebrane [2], [18], [19]. The **DS** lagrangian obtained in [20], [21] corresponds to the supersymmetrised version of $\sim \alpha'$ anomaly cancelling Green-Schwarz (GS) corrections [25] to the simple (lowest α'-order) D=10, N=1 supergravity considered in [24]. The problem with the standard supergravity is that $\sim \alpha'$ corrections can be made supersymmetric only in the same $\sim \alpha'$ order. For complete supersymmetrisation one must take into account the infinite number of terms $\sim \alpha'^n$, $(n = 1, 2, \ldots)$, containing the axionic field and dictated by supersymmetry. Situation is different in the dual supergravity. If the same corrections are expressed in terms of fivebrane variables - the result becomes exactly supersymmetric in the order $\sim \alpha'$, i.e. the infinite series in α' is transformed to the finite number of terms in the case of dual supergravity.

N=1, D=10 **DS** contains 6-form axionic potential $C^{(6)}$, so that 5-brane solution, which was constructed for the lagrangian of [21] in [22], is of *electric* type.

In this paper we obtain a solution of anomaly-free theory, which corresponds to the following situation: one 5-brane is in 012345, it is defined by $X^6 = X^7 = X^8 = X^9 = 0$, the other 5-brane is in 012367 and has $X^4 = X^5 = X^8 = X^9 = 0$. The Lorentz symmetry becomes $SO(1,3) \times SO(2) \times SO(2) \times SO(2)$.

The solution is a vacuum configuration in the sense that all fields depend only on coordinates X^8, X^9, which are transverse to both branes, and all fermions are set to zero.

2 Lagrangian

The lagrangian of N=1, D=10 **DS** is equal to:

$$\mathcal{L} = \mathcal{L}^{(gauge)} + \mathcal{L}^{(grav)}, \tag{2}$$

where $\mathcal{L}^{(gauge)}$ and $\mathcal{L}^{(grav)}$ are lagrangians for gauge-matter and for supergravity multiplet.

We consider only bosonic terms in the lagrangian (2) because we are interested in the vacuum configuration. Notations correspond in general to [20]. We use the following index notations for flat indices: $\hat{A} = (\alpha,\, A,\, \tilde{A},\, a)$, where

$\hat{A} = 0, \ldots, 9;\;\; \alpha = 0, \ldots, 3;\; A = 4, 5;\quad \tilde{A} = 6, 7;\;\; a = 8, 9$ and for world indices:

$$X^{\hat{M}} = (x^\mu,\, x^M,\, x^{\tilde{M}},\, y^m),$$

$\hat{M} = 0, \ldots, 9;\;\; \mu = 0, \ldots, 3;\; M = 4, 5;\quad \tilde{M} = 6, 7;\;\; m = 8, 9.$

We present the gravity part of the lagrangian as an expansion in α':

$$\mathcal{L}^{(grav)} = \mathcal{L}_0^{(grav)} + \alpha' \mathcal{L}_1^{(grav)}, \tag{3}$$

where $\mathcal{L}_0^{(grav)}$ is equal to [23] (see [21] for further references on the subject):

$$2\, k^2\, E^{-1}\, \mathcal{L}_0^{(grav)} = \phi \left(R - \frac{1}{12}\, M^2_{\hat{A}\hat{B}\hat{C}} \right). \tag{4}$$

R is the curvature scalar, ϕ is the dilatonic field, $M_{\hat{A}\hat{B}\hat{C}}$ is defined by:

$$M_{\hat{A}\hat{B}\hat{C}} = E^{\hat{P}}_{\hat{A}} E^{\hat{Q}}_{\hat{B}} E^{\hat{R}}_{\hat{C}} M_{\hat{P}\hat{Q}\hat{R}}, \quad M^{(3)} = {}^{\star}dC^{(6)}, \tag{5}$$

where $E^{\hat{A}}_{\hat{M}}$ - tenbein.

The result for $\mathcal{L}_1^{(grav)}$ was obtained in [20], [21] in the form:

$$\begin{aligned} 2\,k^2\,E^{-1}\,\mathcal{L}_1^{(grav)} &= 2\,R^2_{\hat{A}\hat{B}} - R^2_{\hat{A}\hat{B}\hat{C}\hat{D}} + \frac{1}{2\cdot 6!}\varepsilon^{\hat{A}\hat{B}\hat{C}\hat{D}\hat{F}_1\ldots\hat{F}_6}\,R_{\hat{A}\hat{B}}{}^{\hat{I}\hat{J}}R_{\hat{C}\hat{D}\hat{I}\hat{J}}C_{\hat{F}_1\ldots\hat{F}_6} - \\ &-\frac{1}{2}\,R^{\hat{A}\hat{B}}(M^2)_{\hat{A}\hat{B}} - \frac{1}{6}\,M^{\hat{A}\hat{B}\hat{C}}D^2_{\hat{F}}\,M_{\hat{A}\hat{B}\hat{C}} + \\ &+\frac{1}{2}\,M^{\hat{A}\hat{B}\hat{C};\hat{D}}M^2_{\hat{A}\hat{B}\hat{C}\hat{D}} - \frac{1}{24}\,(M^2)_{\hat{A}\hat{B}\hat{C}\hat{D}}M^2_{\hat{A}\hat{C}\hat{B}\hat{D}}. \end{aligned} \tag{6}$$

$R_{\hat{A}\hat{B}\hat{C}\hat{D}}$ is the curvature tensor, $R_{\hat{A}\hat{B}}$ is the Richi tensor, $(;\hat{B})$ means the covariant derivative $D_{\hat{B}}$ and the following notations are introduced:

$$M^2 = (M_{\hat{A}\hat{B}\hat{C}})^2, \;\; (M^2)_{\hat{A}\hat{B}} = M_{\hat{A}}{}^{\hat{C}\hat{D}}M_{\hat{B}\hat{C}\hat{D}}$$
$$(M^2)_{\hat{A}\hat{B}\hat{C}\hat{D}} = M_{\hat{A}\hat{B}}{}^{\hat{F}}M_{\hat{C}\hat{D}\hat{F}}, \;\; (M^3)_{\hat{A}\hat{B}\hat{C}} = M_{\hat{A}}{}^{\hat{I}\hat{J}}M_{\hat{B}\hat{J}}{}^{\hat{K}}M_{\hat{C}\hat{K}\hat{I}}.$$

We need not $\mathcal{L}^{(gauge)}$ in this paper, since we discuss a vacuum solution with zero gauge-matter (it is shown bellow).

One can make fields redefinition :

$$\phi = e^{\frac{2}{3}\varphi}, \quad g_{\hat{M}\hat{N}} = e^{-\frac{1}{6}\varphi}\,g_{\hat{M}\hat{N}}{}^{(can)}, \quad C^{(6)} = C^{(can)\,(6)} \tag{7}$$

to obtain lagrangian with canonical kinetic terms (see for details [21]).

3 Vacuum Configuration

Let us study equations defining vacuum configuration: $<\delta_Q\,\Phi>= 0$, where Φ is some field but δ_Q is a supersymmetry transformation. When Φ is a boson, such an equation is satisfied identically. It has nontrivial content for $\Phi = \psi_A$, χ, λ, i.e. for gravitino, dilatino and gaugino fields respectively.

We work with formulas from [21] (see also [26], [27] where another parametrization is used):

$$<\delta_Q\psi_{\hat{A}}>= \epsilon_{;\hat{A}} + \frac{1}{144}\left(3\,M_{\hat{B}\hat{C}\hat{D}}\Gamma^{\hat{B}\hat{C}\hat{D}}\Gamma_{\hat{A}} + \Gamma_{\hat{A}}M_{\hat{B}\hat{C}\hat{D}}\Gamma^{\hat{B}\hat{C}\hat{D}}\right)\epsilon = 0, \tag{8}$$

$$<\delta_Q\chi>= \frac{1}{2}\partial_{\hat{A}}\phi\,\Gamma^{\hat{A}}\epsilon - \left(\frac{\phi}{36}\,M_{\hat{A}\hat{B}\hat{C}}\Gamma^{\hat{A}\hat{B}\hat{C}} - \alpha'\,A_{\hat{A}\hat{B}\hat{C}}\Gamma^{\hat{A}\hat{B}\hat{C}}\right)\epsilon = 0, \tag{9}$$

$$<\delta_Q\lambda>= \frac{1}{4}\mathcal{F}_{\hat{A}\hat{B}}\Gamma^{\hat{A}\hat{B}}\epsilon = 0. \tag{10}$$

Here $\epsilon(y)$ is a 32-component Dirac spinor - the parameter of supersymmetry transformation. It is subjected to the Majorana-Weyl condition: $\epsilon_c = \bar{\epsilon}$, $\epsilon = \Gamma\epsilon$, where Γ is the

chirality matrix, ϵ_c is the charge-conjugated spinor. (Let us recall that all fields depend only on transverse coordinates y^m).

The 3-form field $A_{\hat{A}\hat{B}\hat{C}}$ in eq.(9) is equal to [20],[21]:

$$A_{\hat{A}\hat{B}\hat{C}} = -\frac{1}{18} D_{\hat{F}}{}^2\, M_{\hat{A}\hat{B}\hat{C}} + \frac{7}{36}(M^2_{\hat{D}[\hat{A}\hat{B}\hat{C}]})^{;\hat{D}} + \frac{1}{36} M_{\hat{D}\hat{E}[\hat{A};\hat{B}} M_{\hat{C}]}{}^{\hat{D}\hat{E}} -$$
$$-\frac{5}{8\cdot 243} M^2\, M_{\hat{A}\hat{B}\hat{C}} + \frac{5}{8\cdot 27} M^2_{\hat{D}[\hat{A}} M_{\hat{B}\hat{C}]}{}^{\hat{D}} - \frac{5}{4\cdot 27} M^3_{\hat{A}\hat{B}\hat{C}} -$$
$$- \frac{1}{4\cdot 972} \varepsilon_{\hat{A}\hat{B}\hat{C}}{}^{\hat{D}\hat{E}\hat{F}\hat{G}\hat{H}\hat{I}\hat{J}} M_{\hat{D}\hat{E}\hat{F}} (M_{\hat{H}\hat{I}\hat{J};\hat{G}} + M^2_{\hat{G}\hat{H}\hat{I}\hat{J}}). \tag{11}$$

(We also hold on the standard notation: $\Gamma_{\hat{A}_1 \ldots \hat{A}_k} = \Gamma_{[\hat{A}_1} \Gamma_{\hat{A}_2} \ldots \Gamma_{\hat{A}_k]}$.)

We begin our study from eq.(8). It is essential thing, that to find non-trivial solution of eq.(8) one must impose two additional conditions on ϵ:

$$\varepsilon_{bj}\, \Gamma^{45}\, \Gamma^j\, \epsilon = \nu\, \Gamma_b\, \epsilon, \quad \nu^2 = 1, \quad \varepsilon_{bj}\, \Gamma^{67}\, \Gamma^j\, \epsilon = \tilde{\nu}\, \Gamma_b\, \epsilon, \quad \tilde{\nu}^2 = 1, \tag{12}$$

thus keeping $\frac{1}{4}$ of initial N=1, D=10 supersymmetry. We conclude that our vacuum possesses N=1, D=4 supersymmetry.

Then, using the following anzats for non-zero components of tenbein:

$$E_{\hat{M}}{}^{\hat{A}} = \begin{pmatrix} e^{\xi_1(y)}\, \delta^\alpha_\mu & & & \\ & e^{\xi_2(y)}\, \delta^A_M & & \\ & & e^{\xi_3(y)}\, \delta^{\tilde{A}}_{\tilde{M}} & \\ & & & e^{\xi_4(y)}\, \delta^a_m \end{pmatrix} \tag{13}$$

and of axionic potential:

$$C^{(6)} = \tilde{\lambda}\, e^{H_1(y)}\, dx^0 \wedge \ldots \wedge dx^3 \wedge dx^4 \wedge dx^5 + \lambda\, e^{H_2(y)}\, dx^0 \wedge \ldots \wedge dx^3 \wedge dx^6 \wedge dx^7, \tag{14}$$

one can obtain from eq.(8):

$$\xi_1 = \frac{1}{6}\,(H_1 + H_2), \qquad \xi_2 = \frac{1}{6}\, H_1 - \frac{1}{3}\, H_2, \tag{15}$$

$$\xi_3 = \frac{1}{6}\, H_2 - \frac{1}{3}\, H_1, \qquad \xi_4 = -\frac{1}{3}\,(H_1 + H_2), \tag{16}$$

$$\lambda = \nu = \pm 1, \quad \tilde{\lambda} = \tilde{\nu} = \pm 1, \quad \epsilon(y) = e^{-\frac{\xi_4}{4}}\, \epsilon^0, \tag{17}$$

where ϵ^0 is a constant spinor.

Anzats (13), (14) with account for relations (15)-(17) leads to the following non-zero components of $M^{(3)}$-form :

$$M_{\tilde{P}\tilde{Q}r} = \tilde{\nu}\, \varepsilon_{\tilde{P}\tilde{Q}}\, \delta^i_r\, \varepsilon_{ij}\, H_1{}^j, \qquad M_{PQr} = \nu\, \varepsilon_{PQ}\, \delta^i_r\, \varepsilon_{ij}\, H_2{}^j, \tag{18}$$

where we introduce the notations:

$$H_1{}^j = \eta^{jm}\, \partial_m\, H_1, \quad H_2{}^j = \eta^{jm}\, \partial_m\, H_2, \;\; \eta^{jm} = -\delta^{jm} \tag{19}$$

and

$$\varepsilon_{PQ} = e^{2\,\xi_2}\,\varepsilon_{AB}\,\delta^A_P\,\delta^B_Q, \quad \varepsilon_{\tilde{P}\tilde{Q}} = e^{2\,\xi_3}\,\varepsilon_{\tilde{A}\tilde{B}}\,\delta^{\tilde{A}}_{\tilde{P}}\,\delta^{\tilde{B}}_{\tilde{Q}}. \tag{20}$$

We give formulas for surviving components of 10D-spin-connection and curvature tensor in Appendix.

Now we turn to eq.(10), which reduces for our vacuum to $\mathcal{F}_{ab}\Gamma^{ab}\epsilon = 0$. It follows that $\mathcal{F}_{ab} = 0$.

We are left with eq.(9) which is the most complicated one. Taking into account eq.(18) and formulas from Appendix one is able to calculate:

$$A_{\hat{A}\hat{B}\hat{C}}\,\Gamma^{\hat{A}\hat{B}\hat{C}}\,\epsilon = -\left[e^{-3\xi_4}\,({\xi_4}^f{}_f - \frac{1}{2}\,{\xi_4}^f\,\xi_{4f})\right]^j\,\Gamma_j\,\epsilon. \tag{21}$$

Using (21) one can drop one derivative in eq.(9) and find the expression for dilaton:

$$\phi = C_0\,e^{\xi_4} + 2\,\alpha'\,C_0\,e^{-2\xi_4}\,({\xi_4}^f{}_f - \frac{1}{2}\,{\xi_4}^f\,\xi_{4f}), \tag{22}$$

where C_0 is an arbitrary constant.

At this stage of our consideration we expressed all fields in terms of two arbitrary functions H_1 and H_2.

We examine bellow whether equations of motion impose constraints on these functions.

4 Equations of Motion

Equations for N=1, D=10 **DS** obtained in the lowest and next order in α' are used (see [20], [21]), see also [28] where another parametrization was considered).

We start from axion EM, which can be written in the form [20]:

$$\mathcal{H}_{[\hat{A}\hat{B}\hat{C};\hat{D}]} = 3\,\alpha' R_{[\hat{A}\hat{B}}{}^{\hat{E}\hat{F}}R_{\hat{C}\hat{D}]\hat{F}\hat{E}}, \tag{23}$$

where

$$\mathcal{H}_{\hat{A}\hat{B}\hat{C}} = \phi\,M_{\hat{A}\hat{B}\hat{C}} - 2\,\alpha'\Big(-D_{\hat{F}}{}^2\,M_{\hat{A}\hat{B}\hat{C}} + 3\,(M^2_{\hat{D}[\hat{A}\hat{B}\hat{C}]})^{;\hat{D}} + \\ + \frac{3}{2}M_{\hat{D}\hat{F}[\hat{A};\hat{B}}M_{\hat{C}]}{}^{\hat{D}\hat{F}} - 3\,R_{\hat{D}[\hat{A}}M_{\hat{B}\hat{C}]}{}^{\hat{D}} - \frac{1}{2}M^3_{[\hat{A}\hat{B}\hat{C}]}\Big). \tag{24}$$

For our configuration we have: $R_{[\hat{A}\hat{B}}{}^{\hat{E}\hat{F}}R_{\hat{C}\hat{D}]\hat{F}\hat{E}} = 0$ and only the $\mathcal{H}_{ABc}$ and $\mathcal{H}_{\tilde{A}\tilde{B}c}$ components survive:

$$\mathcal{H}_{ABc} = {\mathcal{H}_{ABc}}^{(0)} + \alpha'\,{\mathcal{H}_{ABc}}^{(1)},$$

$$\begin{aligned} {\mathcal{H}_{ABc}}^{(0)} &= \nu\,C_0\,\varepsilon_{AB}\varepsilon_{cj}\,H_2^j, \\ {\mathcal{H}_{ABc}}^{(1)} &= \nu\,\varepsilon_{AB}\varepsilon_{cj}\,e^{-3\xi_4}\,\Big(2\,{H_2}^{jf}{}_f - \frac{2}{3}\,{H_1}^j{}_f\,{H_2}^f + \frac{2}{3}\,{H_2}^j{}_f\,{H_1}^f + \\ &+\frac{4}{3}\,{H_2}^f{}_f\,{H_1}^j + \frac{2}{3}\,{H_1}^f{}_f\,{H_2}^j + \frac{2}{3}\,{H_2}^j\,({H_1}^f\,H_{1f}) - \frac{2}{3}\,{H_1}^j\,({H_1}^f\,H_{2f})\Big). \end{aligned} \tag{25}$$

$\mathcal{H}_{\tilde{A}\tilde{B}c}$ can be written from (25) with the following interchange:

$$\varepsilon_{AB} \to \varepsilon_{\tilde{A}\tilde{B}}, \quad \nu \to \tilde{\nu}, \quad H_2 \leftrightarrow H_1.$$

Using (25) and (44), (45) axionic EM (eq. 23) results in 2 equations:

$$\varepsilon_{AB}\, \varepsilon_{cd} \left(e^{-H_2}\, C_0 - 2\,\alpha'\, H_{2f}{}^{f}\, e^{H_1}\right)^{j}{}_{j} = 0, \tag{26}$$

$$\varepsilon_{\tilde{A}\tilde{B}}\, \varepsilon_{cd} \left(e^{-H_1}\, C_0 - 2\,\alpha'\, H_{1f}{}^{f}\, e^{H_2}\right)^{j}{}_{j} = 0. \tag{27}$$

Let's go to the dilaton equation of motion. It can be presented in the form:

$$\begin{aligned} &D_{\hat{F}}{}^2\phi + \frac{1}{12}\phi M^2_{\hat{A}\hat{B}\hat{C}} - \alpha' \left[-\frac{2}{3}(R_{\hat{A}\hat{B}})^2 + \frac{1}{3}(R_{\hat{A}\hat{B}\hat{C}\hat{D}})^2 - \right.\\ &-\frac{1}{6}R^{\hat{A}\hat{B}}M^2_{\hat{A}\hat{B}} - \frac{1}{18}M^{\hat{A}\hat{B}\hat{C}}D_{\hat{F}}{}^2 M_{\hat{A}\hat{B}\hat{C}} + \frac{1}{3}M^{\hat{A}\hat{B}\hat{C};\hat{D}}M^2_{\hat{A}\hat{B}\hat{C}\hat{D}} - \\ &\left.-\frac{1}{24}M^2_{\hat{A}\hat{B}\hat{C}\hat{D}}M^{2\hat{A}\hat{C}\hat{B}\hat{D}} - \frac{1}{12}D_{\hat{F}}{}^2 M^2 + \frac{1}{6}(M^2_{\hat{A}\hat{B}})^{;\hat{A}\hat{B}}\right] = 0. \end{aligned} \tag{28}$$

Calculation of all the terms in eq.(28) with the help of relations obtained before gives:

$$e^{H_1}\left(e^{-H_1}\, C_0 - 2\,\alpha'\, H_{1f}{}^{f}\, e^{H_2}\right)^{j}{}_{j} + e^{H_2}\left(e^{-H_2}\, C_0 - 2\,\alpha'\, H_{2f}{}^{f}\, e^{H_1}\right)^{j}{}_{j} = 0, \tag{29}$$

which is satisfied simultaneously with eq.(26), (27).

A study of a rather complicated graviton equation of motion ($\phi R_{\hat{A}\hat{B}} + \ldots = 0$) does not produce additional constraints for vacuum configuration.

5 The choice of the solution

One must solve eq.(26), (27), matching the singular sources, the 1^{st} brane being the source for eq.(27), the 2^{d} - for eq.(26). We want to discuss the solutions of the two types:

1. After introducing complex variable $z = y^8 + i\, y^9$, one can easily see, that equations (26), (27) are automatically satisfied for arbitrary analytical functions:

   $H_1 = H_1(z)$, $H_2 = H_2(z)$. Moreover, all α' corrections also vanish in this case and we have: $\phi = e^{-\frac{1}{3}(H_1+H_2)}$, putting here and bellow $C_0 = 1$,

   $$\mathcal{H}_{67z} = \phi\, M_{67z} = i\,\tilde{\nu}\partial_z\,(e^{-H_1}), \quad \mathcal{H}_{67\bar{z}} = 0, \quad \mathcal{H}_{45z} = \phi\, M_{45z} = i\,\nu\partial_z\,(e^{-H_2}), \quad \mathcal{H}_{45\bar{z}} = 0,$$

   where $M_{PQz} = \frac{1}{2}\,(M_{PQ8} - i\, M_{PQ9})$, etc. We ought to choose:

   $$e^{-H_1} = C_1 - q_1\, \ln\frac{z}{\rho}, \quad e^{-H_2} = C_2 - q_2\, \ln\frac{z}{\rho}, \;\; q_1 > 0, \;\; q_2 > 0, \;\; C_1 > 0, \;\; C_2 > 0 \tag{30}$$

   to obtain non-zero charges:

   $$Q_1 = \frac{1}{2\,\pi}\int\limits_{|z|^2=\rho^2} \mathcal{H}_{67z}dz = \tilde{\nu}\, q_1, \qquad Q_2 = \frac{1}{2\,\pi}\int\limits_{|z|^2=\rho^2} \mathcal{H}_{45z}dz = \nu\, q_2. \tag{31}$$

Note that integration of $\mathcal{H}^{(3)}$ over 3-sphere $x_6^2+x_7^2+|z|^2=\rho^2$ reduces to integration over circle $|z|^2=\rho^2$, since $\mathcal{H}_{\tilde{P}\tilde{Q}r}$ is in fact $\sim \delta(x_6)\,\delta(x_7)$. (Remember that $1^{\underline{st}}$ brane is situated in the origin of 6,7-coordinates.) Analogously $\mathcal{H}_{PQr} \sim \delta(x_4)\,\delta(x_5)$.

The explicit formulas for dilaton and axion of the constructed solution are:

$$\phi = \Big(C_1 - q_1\,\ln\frac{z}{\rho}\Big)^{\frac{1}{3}}\,\Big(C_2 - q_2\,\ln\frac{z}{\rho}\Big)^{\frac{1}{3}},$$

$$C_{012345} = \tilde{\nu}\Big(C_1 - q_1\,\ln\frac{z}{\rho}\Big)^{-1}, \quad C_{012367} = \nu\Big(C_2 - q_2\,\ln\frac{z}{\rho}\Big)^{-1}. \tag{32}$$

(We must demand $|z| < \rho$ to avoid singularities at $z \neq 0$).

The only non-zero component of Richi-tensor is:

$$R_{zz} = \frac{1}{9\,z^2}\left[\Big(q_1\,e^{H_1} + q_2\,e^{H_2}\Big)^2 + \frac{3}{2}\,q_1\,e^{H_1}\,\Big(5q_1\,e^{H_1} - 2\Big) + \frac{3}{2}\,q_2\,e^{H_2}\,\Big(5q_2\,e^{H_2} - 2\Big)\right]. \tag{33}$$

Curvature scalar R and lagrangian (eq.2) are zero on this solution (it is true for any $H_1(z)$ and $H_2(z)$).

It is worth emphasizing, that if we had neglected α' corrections, then the general solution would have been:

$$e^{-H_1} = C_1 - q_1\,\ln\frac{z}{\rho} - q_1{}'\,\ln\frac{\bar{z}}{\rho}, \quad q_1 \geq 0, q_1{}' \geq 0; \tag{34}$$

$$e^{-H_2} = C_2 - q_2\,\ln\frac{z}{\rho} - q_2{}'\,\ln\frac{\bar{z}}{\rho}, \quad q_2 \geq 0, q_2{}' \geq 0. \tag{35}$$

If one of the q_1, $q_1{}'$ and one of the q_2, $q_2{}'$ is set to zero, the choice (34), (35) goes through eq.(26), (27) without any modifications, but this is not the case if $q_1 \neq 0$, $q_1{}' \neq 0$ or $q_2 \neq 0$, $q_2{}' \neq 0$.

2. Let us analyze the solution of the type $H_1(r)$, $H_2(r)$, possessing SO(2) symmetry in z-plane, here $r = |z|^2$. We integrate eq.(26), (27) twicely and write:

$$e^{-H_1} - 2\,\alpha'\,H_{1f}{}^{f}\,e^{H_2} = C_1 - q_1\,\ln\frac{r}{\rho}, \tag{36}$$

$$e^{-H_2} - 2\,\alpha'\,H_{2f}{}^{f}\,e^{H_1} = C_2 - q_2\,\ln\frac{r}{\rho}. \tag{37}$$

One can see, that in the vicinity of $r=0$ the solution of this system behaves as :

$$e^{-H_1} \sim -\sqrt{2\,\alpha'}\,\Big(r\ln\frac{r}{\rho}\Big)^{-1}, \quad e^{-H_2} \sim -\sqrt{2\,\alpha'}\,\Big(r\ln\frac{r}{\rho}\Big)^{-1}. \tag{38}$$

In the limit $r \to 0$ it is more singular than $\ln\frac{r}{\rho}$, which would have described the behaviour of the solution, if we had neglected terms with α' in (36), (37). For (standard) dilaton and curvature scalar in $r \to 0$ limit we obtain:

$$e^{2\,\varphi} = \frac{4^2\,\alpha'}{9^3\,r^2}\,\Big(\ln\frac{r}{\rho}\Big)^4, \qquad R = -\frac{1}{r^2}. \tag{39}$$

Far from the origin one can solve eq.(36), (37) by expanding in α' (assuming $\alpha' \ln\Big(\frac{r}{\rho}\Big) \ll 1$) and using $q_1 \sim \alpha'$, $q_2 \sim \alpha'$ (see [4], [8]), so that the expressions for the defining functions are:

$$e^{-H_1} = C_1 - q_1 \ln\frac{r}{\rho} - 2\alpha' \frac{q_1^2}{C_1^2\, C_2\, r^2} + O\Big(\alpha'^4\Big), \quad C_1 > 0; \tag{40}$$

$$e^{-H_2} = C_2 - q_2 \ln\frac{r}{\rho} - 2\alpha' \frac{q_2^2}{C_2^2\, C_1\, r^2} + O\Big(\alpha'^4\Big), \quad C_2 > 0. \tag{41}$$

The SO(2) symmetric solution has charges:

$$Q_1 = \frac{1}{2\,\pi} \int\limits_{|z|^2=\rho^2} \Big(-\tilde{\nu}\partial_r\, e^{-H_1}\Big)\, dz = \tilde{\nu}\, q_1, \quad Q_2 = \frac{1}{2\,\pi} \int\limits_{|z|^2=\rho^2} \Big(-\nu\partial_r\, e^{-H_2}\Big) dz = \nu\, q_2. \tag{42}$$

Here only $\mathcal{H}^{(0)}_{67m}$ ($\mathcal{H}^{(0)}_{45m}$) contributes to Q_1 (Q_2), since $\mathcal{H}^{(1)}_{67m}$, $\mathcal{H}^{(1)}_{45m}$ vanish at infinity more rapidly than $\frac{1}{r}$.

We give also the formula for dilaton up to $O\Big(\alpha'^4\Big)$:

$$e^{2\,\varphi} = C_1\, C_2 - (q_1\, C_2 + q_2\, C_1)\, \ln\frac{r}{\rho} + q_1\, q_2 \left(\ln\frac{r}{\rho}\right)^2 + \frac{\alpha'}{3\, r^2}\left(\frac{q_1}{C_1} + \frac{q_2}{C_2}\right)^2. \tag{43}$$

6 Conclusion

Let's summarize our results. We've constructed the solution of anomaly-free N=1, D=10 Dual Supergravity, which corresponds to the presence of the two intersecting 5-branes. We've found that accounting for higher-order-derivative (up to the $4^{(th)}$) terms in the Lagrangian bring forth modifications of the solution. An important thing is that the two functions H_1 and H_2, determing all the fields, are mixed in eq.(26), (27). Let us recall that the solutions of Lagrangians with up to the 2^d order-derivative-terms, widely discussed in literature, are characterized by the independent harmonic functions ([12], [13], [15]). e^{-H_1}, e^{-H_2} would have played the role of these functions in the solution without α' corrections,

In the SO(2) symmetric case modifications display themselves not only in α' corrections to the solution far from the origin, but also in changing the type of singular behaviour in the vicinity of $r = 0$.

Remarkably, analytic solution is exact: field configuration doesn't acquire α' modifications and H_1, H_2 remain independent.

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7 Appendix

Here are the formulas for non-zero components of 10D-spin-connection:

$$\omega_{\alpha\beta}{}^{c} = -\, e^{-\xi_4}\, \eta_{\alpha\beta}\, \xi_1{}^{c}, \quad \omega_{AB}{}^{c} = -e^{-\xi_4}\, \eta_{AB}\, \xi_2{}^{c}, \quad \omega_{\tilde{A}\tilde{B}}{}^{c} = -e^{-\xi_4}\, \eta_{\tilde{A}\tilde{B}}\, \xi_3{}^{c}, \tag{44}$$

$$\omega_{abc} = -2e^{-\xi_4}\, \eta_{a[b}\, \xi_{2c]}, \tag{45}$$

where $\eta_{\hat{A}\hat{B}} = (1,-1,-1,-1,-1,-1,-1,-1,-1,-1)$.

The folowing curvature tensor componets survive ($R = d\omega + \omega \wedge \omega$):

$$\begin{gathered}
R_{\alpha\beta}{}^{\gamma\delta} = 2\, e^{-2\,\xi_4}\, \delta^{\gamma}_{[\alpha}\delta^{\delta}_{\beta]}\, \xi_1{}^{f}\xi_{1f} \quad R_{AB}{}^{CD} = 2\, e^{-2\,\xi_4}\, \delta^{C}_{[A}\delta^{D}_{B]}\, \xi_2{}^{f}\xi_{2f} \\
R_{\alpha A}{}^{\beta B} = e^{-2\,\xi_4}\, \delta^{\beta}_{\alpha}\delta^{B}_{A}\, \xi_1{}^{f}\xi_{2f} \quad R_{\tilde{A}\tilde{B}}{}^{\tilde{C}\tilde{D}} = 2\, e^{-2\,\xi_4}\, \delta^{\tilde{C}}_{[\tilde{A}}\delta^{\tilde{D}}_{\tilde{B}]}\, \xi_3{}^{f}\xi_{3f} \\
R_{\alpha\tilde{A}}{}^{\beta\tilde{B}} = e^{-2\,\xi_4}\, \delta^{\beta}_{\alpha}\delta^{\tilde{B}}_{\tilde{A}}\, \xi_1{}^{f}\xi_{3f} \quad R_{A\tilde{B}}{}^{C\tilde{D}} = e^{-2\,\xi_4}\, \delta^{C}_{A}\delta^{\tilde{D}}_{\tilde{B}}\, \xi_2{}^{f}\xi_{3f} \\
R_{\alpha b}{}^{\gamma c} = e^{-2\,\xi_4}\delta^{\gamma}_{\alpha}\, (\xi_{1b}{}^{c} - \xi_{1b}\xi_4{}^{c} - \xi_{4b}\xi_1{}^{c} + \xi_{1b}\xi_1{}^{c} + \delta^{c}_{b}\, \xi_1{}^{f}\xi_{4f}) \\
R_{Ab}{}^{Cd} = e^{-2\,\xi_4}\delta^{C}_{A}\, (\xi_{2b}{}^{d} - \xi_{2b}\xi_4{}^{d} - \xi_{4b}\xi_2{}^{d} + \xi_{2b}\xi_2{}^{d} + \delta^{d}_{b}\, \xi_2{}^{f}\xi_{4f}) \\
R_{\tilde{A}b}{}^{\tilde{C}d} = e^{-2\,\xi_4}\delta_{\tilde{A}}{}^{\tilde{C}}\, (\xi_{3b}{}^{d} - \xi_{3b}\xi_4{}^{d} - \xi_{4b}\xi_3{}^{d} + \xi_{3b}\xi_3{}^{d} + \delta^{d}_{b}\, \xi_3{}^{f}\xi_{4f}) \\
R_{ab}{}^{cd} = e^{-2\xi_4}\, (4\, \delta_{[a}{}^{[c}\, \xi_{4b]}{}^{d]} - 4\, \delta_{[a}{}^{[c}\, \xi_{4b]}\xi_4{}^{d]} + 2\, \delta^{c}_{[a}\delta^{d}_{b]}\, \xi_4{}^{f}\xi_{4f})
\end{gathered} \tag{46}$$

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